Chapter 3: Central Tendency
Central Tendency

• In general terms, **central tendency** is a statistical measure that determines a single value that accurately describes the center of the distribution and represents the entire distribution of scores.

• The goal of central tendency is to identify the single value that is the best representative for the entire set of data.
Central Tendency (cont'd.)

• By identifying the "average score," central tendency allows researchers to summarize or condense a large set of data into a single value.

• Thus, central tendency serves as a descriptive statistic because it allows researchers to describe or present a set of data in a very simplified, concise form.

• In addition, it is possible to compare two (or more) sets of data by simply comparing the average score (central tendency) for one set versus the average score for another set.
The Mean, the Median, and the Mode

- It is essential that central tendency be determined by an objective and well-defined procedure so that others will understand exactly how the "average" value was obtained and can duplicate the process.
- No single procedure always produces a good, representative value. Therefore, researchers have developed three commonly used techniques for measuring central tendency: the mean, the median, and the mode.
The Mean

• The mean is the most commonly used measure of central tendency.
• Computation of the mean requires scores that are numerical values measured on an interval or ratio scale.
• The mean is obtained by computing the sum, or total, for the entire set of scores, then dividing this sum by the number of scores.
The Mean (cont'd.)

• Conceptually, the mean can also be defined in the following ways:
  1. The mean is the amount that each individual receives when the total ($\sum X$) is divided equally among all $N$ individuals.
  2. The mean is the balance point of the distribution because the sum of the distances below the mean is exactly equal to the sum of the distances above the mean.
Changing the Mean

- Because the calculation of the mean involves every score in the distribution, changing the value of any score will change the value of the mean.
- Modifying a distribution by discarding scores or by adding new scores will usually change the value of the mean.
- To determine how the mean will be affected for any specific situation you must consider: 1) how the number of scores is affected, and 2) how the sum of the scores is affected.
Changing the Mean (cont'd.)

• If a constant value is added to every score in a distribution, then the same constant value is added to the mean. Also, if every score is multiplied by a constant value, then the mean is also multiplied by the same constant value.
When the Mean Won’t Work

- Although the mean is the most commonly used measure of central tendency, there are situations where the mean does not provide a good, representative value, and there are situations where you cannot compute a mean at all.

- When a distribution contains a few extreme scores (or is very skewed), the mean will be pulled toward the extremes (displaced toward the tail). In this case, the mean will not provide a "central" value.
When the Mean Won’t Work (cont'd.)

• With data from a nominal scale it is impossible to compute a mean, and when data are measured on an ordinal scale (ranks), it is usually inappropriate to compute a mean.

• Thus, the mean does not always work as a measure of central tendency and it is necessary to have alternative procedures available.
The Median

- If the scores in a distribution are listed in order from smallest to largest, the median is defined as the midpoint of the list.
- The median divides the scores so that 50% of the scores in the distribution have values that are equal to or less than the median.
- Computation of the median requires scores that can be placed in rank order (smallest to largest) and are measured on an ordinal, interval, or ratio scale.
The Median (cont'd.)

• Usually, the median can be found by a simple counting procedure:
  1. With an odd number of scores, list the values in order, and the median is the middle score in the list.
  2. With an even number of scores, list the values in order, and the median is half-way between the middle two scores.
The Median (cont'd.)

• If the scores are measurements of a continuous variable, it is possible to find the median by first placing the scores in a frequency distribution histogram with each score represented by a box in the graph.

• Then, draw a vertical line through the distribution so that exactly half the boxes are on each side of the line. The median is defined by the location of the line.
(a) 

(b) 

Median = 3.75
The Median (cont'd.)

• One advantage of the median is that it is relatively unaffected by extreme scores.

• Thus, the median tends to stay in the "center" of the distribution even when there are a few extreme scores or when the distribution is very skewed. In these situations, the median serves as a good alternative to the mean.
The Mode

- The mode is defined as the most frequently occurring category or score in the distribution.
- In a frequency distribution graph, the mode is the category or score corresponding to the peak or high point of the distribution.
- The mode can be determined for data measured on any scale of measurement: nominal, ordinal, interval, or ratio.
The Mode (cont'd.)

• The primary value of the mode is that it is the only measure of central tendency that can be used for data measured on a nominal scale. In addition, the mode often is used as a supplemental measure of central tendency that is reported along with the mean or the median.
Bimodal Distributions

• It is possible for a distribution to have more than one mode. Such a distribution is called bimodal. (Note that a distribution can have only one mean and only one median.)

• In addition, the term "mode" is often used to describe a peak in a distribution that is not really the highest point. Thus, a distribution may have a major mode at the highest peak and a minor mode at a secondary peak in a different location.
Central Tendency and the Shape of the Distribution

• Because the mean, the median, and the mode are all measuring central tendency, the three measures are often systematically related to each other.

• In a symmetrical distribution, for example, the mean and median will always be equal.
Central Tendency and the Shape of the Distribution (cont'd.)

• If a symmetrical distribution has only one mode, the mode, mean, and median will all have the same value.

• In a skewed distribution, the mode will be located at the peak on one side and the mean usually will be displaced toward the tail on the other side.

• The median is usually located between the mean and the mode.
Reporting Central Tendency in Research Reports

• In manuscripts and in published research reports, the sample mean is identified with the letter M.

• There is no standardized notation for reporting the median or the mode.

• In research situations where several means are obtained for different groups or for different treatment conditions, it is common to present all of the means in a single graph.
Reporting Central Tendency in Research Reports (cont'd.)

- The different groups or treatment conditions are listed along the horizontal axis and the means are displayed by a bar or a point above each of the groups.
- The height of the bar (or point) indicates the value of the mean for each group. Similar graphs are also used to show several medians in one display.
Chapter 4: Variability
Variability

• The goal for variability is to obtain a measure of how spread out the scores are in a distribution.
• A measure of variability usually accompanies a measure of central tendency as basic descriptive statistics for a set of scores.
Central Tendency and Variability

• Central tendency describes the central point of the distribution, and variability describes how the scores are scattered around that central point.
• Together, central tendency and variability are the two primary values that are used to describe a distribution of scores.
Variability

• Variability serves both as a descriptive measure and as an important component of most inferential statistics.
• As a descriptive statistic, variability measures the degree to which the scores are spread out or clustered together in a distribution.
• In the context of inferential statistics, variability provides a measure of how accurately any individual score or sample represents the entire population.
Variability (cont'd.)

• When the population variability is small, all of the scores are clustered close together and any individual score or sample will necessarily provide a good representation of the entire set.

• On the other hand, when variability is large and scores are widely spread, it is easy for one or two extreme scores to give a distorted picture of the general population.
Data from Experiment A

- Treatment 1: $M = 35$
- Treatment 2: $M = 40$

Data from Experiment B

- Treatment 1: $M = 35$
- Treatment 2: $M = 40$
Measuring Variability

• Variability can be measured with
  – The range
  – The standard deviation/variance
• In both cases, variability is determined by measuring distance.
The Range

- The **range** is the total distance covered by the distribution, from the highest score to the lowest score (using the upper and lower real limits of the range).
The Range (cont'd.)

• Alternative definitions of range:
  – When scores are whole numbers or discrete variables with numerical scores, the range tells us the number of measurement categories.
  – Alternatively, the range can be defined as the difference between the largest score and the smallest score.
The Standard Deviation

- **Standard deviation** measures the standard (average) distance between a score and the mean.
- The calculation of standard deviation can be summarized as a four-step process:
The Standard Deviation (cont'd.)

1. Compute the deviation (distance from the mean) for each score.
2. Square each deviation.
3. Compute the mean of the squared deviations. For a population, this involves summing the squared deviations (sum of squares, SS) and then dividing by $N$. The resulting value is called the **variance** or **mean square** and measures the average squared distance from the mean.

   For samples, variance is computed by dividing the sum of the squared deviations (SS) by $n - 1$, rather than $N$. The value, $n - 1$, is known as degrees of freedom ($df$) and is used so that the sample variance will provide an unbiased estimate of the population variance.

4. Finally, take the square root of the variance to obtain the standard deviation.
Find the deviation (distance from the mean) for each score

Square each deviation

Find the average of the squared deviations (called "variance")

Take the square root of the variance

Add the deviations and compute the average

The standard deviation or standard distance from the mean

DEAD END
This value is always 0
Properties of the Standard Deviation

• If a constant is added to every score in a distribution, the standard deviation will *not* be changed.

• If you visualize the scores in a frequency distribution histogram, then adding a constant will move each score so that the entire distribution is shifted to a new location.

• The center of the distribution (the mean) changes, but the standard deviation remains the same.
Properties of the Standard Deviation (cont'd.)

• If each score is multiplied by a constant, the standard deviation will be multiplied by the same constant.

• Multiplying by a constant will multiply the distance between scores, and because the standard deviation is a measure of distance, it will also be multiplied.
The Mean and Standard Deviation as Descriptive Statistics

• If you are given numerical values for the mean and the standard deviation, you should be able to construct a visual image (or a sketch) of the distribution of scores.

• As a general rule, about 70% of the scores will be within one standard deviation of the mean, and about 95% of the scores will be within a distance of two standard deviations of the mean.